

Quantum Error Correction Codes with Transversal Clifford Gates

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1 Background

In this project, you will explore problems related to transversal Clifford gates and investigate quantum codes that achieve specific transversal Clifford operations. The study of quantum codes and transversal gates is crucial for quantum fault-tolerant computation, which is a fundamental requirement for building reliable quantum computers. You may refer to Chapter 10 of [NC00] and Chapter 11 of [Got24] for foundational knowledge on this topic.

1.1 Definitions

We introduce two definitions of transversal gates:

Consider a quantum code $\mathcal{C} \subset \mathcal{H}^{\otimes n}$, with an encoding map $C : \mathcal{H} \rightarrow \mathcal{H}^{\otimes n}$ and a projector $\Pi_{\mathcal{C}}$ onto the code space. For any physical quantum state $|\psi\rangle \in \mathcal{H}^{\otimes k}$, its logical counterpart under the code scheme \mathcal{C} is denoted as $|\psi\rangle_L := C^{\otimes k}|\psi\rangle \in (\mathcal{H}^{\otimes k})^{\otimes n}$.

Let \mathcal{G} be the set of all permitted quantum gates on physical qubits. For any quantum gate $G \in \mathcal{G}$, its logical implementation under the code scheme \mathcal{C} is denoted as G_L , satisfying the condition $G_L|\psi\rangle_L = (G|\psi\rangle)_L$ for any physical quantum state $|\psi\rangle$.

We define two types of transversal gates:

1. **Strict Transversality:** A code \mathcal{C} admits a strictly transversal logical gate G_L for $G \in \mathcal{G}$ if $G_L|\psi\rangle_L = G^{\otimes n}|\psi\rangle_L$ for $|\psi\rangle_L$ encoding any k -qubit state $|\psi\rangle$ if G is a gate of k qubits.
2. **Weak Transversality:** A code \mathcal{C} admits a weakly transversal logical gate G_L for $G \in \mathcal{G}$ if $G_L|\psi\rangle_L = G^{\otimes n}|\psi\rangle_L$ for $|\psi\rangle_L$ encoding any k -qubit state $|\psi\rangle$ if G is a gate of k qubits.

Two well-known results are that all stabilizer codes achieve strict transversal Pauli gates, and all CSS codes achieve strict transversal CNOT gates.

Transversal operations are crucial for fault-tolerant quantum computation as they prevent error propagation across code blocks.

2 Problems

This project consists of three tasks, progressing in complexity. The first is relatively straightforward and can be found in [Got16], while the second is research-level and may benefit from insights in [ZCC11]. The third is significantly more challenging; solving it could lead to a publishable result in quantum information theory, and [GG24] provides relevant background.

1. **Strictly Transversal Hadamard and Phase Gates:** Identify quantum codes that achieve strict transversal Hadamard and Phase gates. Derive necessary and sufficient conditions for their existence.
2. **Weakly Transversal Hadamard and Phase Gates:** Identify quantum codes that achieve weak transversal Hadamard and Phase gates. Provide necessary and sufficient conditions, or present them separately if a full characterization is difficult.
3. **QLDPC Codes with Transversal Hadamard Gates:** Investigate whether good QLDPC codes exist that support transversal Hadamard gates (either strict or weak). Provide explicit constructions or proofs of existence.

3 Contacts

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4 References

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