

Quantum Circuits and Polynomials

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1 Background

In this project, you will investigate problems related to the representation of quantum circuits using polynomials. This approach has been extensively studied in the field of quantum computing, with foundational contributions from [DHH+05] and further advancements by [Mon17].

Consider quantum circuits composed of gates such as H, Z, CZ, and CCZ. In [Mon17], it is demonstrated that for any n -qubit quantum circuit C containing h Hadamard gates, there exists a degree-3 polynomial f_C over the finite field of order two. This polynomial satisfies the relationship:

$$\langle 0^n | C | 0^n \rangle = \frac{\text{gap}(f_C)}{2^{n+h/2}},$$

where $\text{gap}(f_C)$ represents the difference between the number of solutions (x such that $f_C(x) = 0$) and the number of non-solutions ($f_C(x) = 1$) of the polynomial f_C .

2 Problems

You are asked to work on two problems: the first is relatively straightforward, while the second is research-level.

1. Can you establish a similar result for circuits using the gate set of H, T, and CNOT? Additionally, can you provide general criteria for gate sets that can be represented in a similar manner?
2. The transformation from a circuit C to its corresponding function f_C is efficient, and each circuit C uniquely determines f_C . However, multiple distinct circuits can correspond to the same function f when given as input. Develop algorithms to find circuits C that minimize the number of qubits required for a given input function f . Alternatively, can you establish any hardness results for this problem?

The challenge lies in the fact that both H and CZ gates correspond to quadratic terms in the function f , making it unclear whether a term

arises from a Hadamard gate or not. The goal is to maximize the use of Hadamard gates, as this reduces the number of qubits needed to represent the polynomial.

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4 References

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